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Noncommutative dipole field theories and unitarity

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Abstract: We extend the argument of Gomis and Mehen for violation of unitarity in field theories with space-time noncommutativity to dipole field theories. In dipole field theories with a timelike dipole vector, we present 1-loop amplitudes that violate the optical theorem. A quantum mechanical system with nonlocal potential of finite extent in time also shows violation of unitarity.

Keywords: Field Theories in Lower Dimensions, Space-Time Symmetries, Non-Commutative Geometry.

Contents

1. Introduction

In a seminal paper three years ago [1] Gomis and Mehen demonstrated that certain temporally nonlocal field theories violate unitarity. The theories studied by Gomis and Mehen were field theories on a noncommutative space-time [2]–[7] and their results agreed with predictions of string theory. In particular, for one such theory — U(N) Yang-Mills theory with $\mathcal{N} = 4$ supersymmetry on a noncommutative spacetime — results from string theory suggested that unitarity can be restored by adding an infinite tower of massive fields [8, 9, 10]. The resulting theory is the well-known "Noncommutative Open String Theory" (NCOS). In fact, this NCOS is dual to spatially noncommutative $\mathcal{N} = 4$ U(N) Yang-Mills theory [8, 9], which is unitary. The dual relation between temporally noncommutative U(1) $\mathcal{N} = 4$ Yang-Mills theory and its spatially noncommutative version can be demonstrated using field theory arguments alone [11].

Building a theory on a noncommutative spacetime is one way to realize nonlocality, but it also introduces extra complications such as the UV/IR relation — the relation between the momentum of a particle and its effective transverse size [12]. This makes the nonlocality length scale increase indefinitely at high momentum [13]. However, within the framework of string theory, it is possible to construct other nonlocal theories where the nonlocality

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scale is bounded. An example is the dipole field theory (DFT) constructed in [14] (see also [15, 16]) and further studied in [17]–[25]. In these theories the nonlocality length is a fixed spacetime vector (the "dipole-vetor"), and DFTs do not exhibit a UV/IR relation. Like the theories built on a noncommutative space, these DFTs break Lorentz invariance. Unlike noncommutative space, however, a suitably constructed DFT can preserve SO(3) rotational invariance. For this to be the case, the dipole vector has to be timelike and the theory will be nonlocal in time.

General theories with timelike nonlocality have been studied in several places (see for instance [26]–[31]) and the question of unitarity was also addressed. Even the simplest example of a harmonic oscillator with timelike nonlocality has complex energy levels. The violation of unitarity in theories on a noncommutative spacetime has also been recently studied in [32].

In this work, following Gomis and Mehen, we demonstrate that timelike DFTs violate unitarity at the 1-loop level. In an upcoming work, we will explore their NCOS completion to a unitary theory [34].

This paper is organized as follows. In section 2, we briefly outline the mathematical backgrounds of DFTs which are originally discussed in [20]. In section 3, based on [20], the noncommutative dipole gauge theory with adjoint matter is formulated and the Feynman rules are derived. Later, in section 4, we show that Feynman diagrams of the theories with timelike dipole vectors do not satisfy the cutting rule and thus the unitarity is violated. Finally, in section 5, interested in 0+1D quantum mechanical systems, we study harmonic oscillators with nonlocal interaction in time and the result gives complex-valued energy levels or complex poles in the propagator, both of which indicate the violation of unitarity. We conclude in section 6 with a discussion of our results and their relation to the limit of string theory.

2. Mathematical backgrounds

In DFTs, we assign a constant *dipole vector* L_i to each field ϕ_i and define the "*dipole star product*" as

$$(\phi_i * \phi_j)(x) \equiv \phi_i\left(x - \frac{1}{2}L_j\right)\phi_j\left(x + \frac{1}{2}L_i\right). \tag{2.1}$$

The noncommutativity of the star product originates from the dipole vector associated to each field. Meanwhile, the requirement that the dipole star product should be associative requires the dipole vector of $(\phi_i * \phi_j)(x)$ to be $L_i + L_j$. The associativity is satisfied via

$$\begin{aligned}(\phi_i * \phi_j) * \phi_k &= \left(\phi_i\left(x - \frac{1}{2}L_j\right)\phi_j\left(x + \frac{1}{2}L_i\right)\right) * \phi_k \\ &= \phi_i\left(x - \frac{L_j + L_k}{2}\right)\phi_i\left(x + \frac{L_i - L_k}{2}\right)\phi_k\left(x + \frac{L_i + L_j}{2}\right) \\ &= \phi_i\left(x - \frac{L_j + L_k}{2}\right)(\phi_j * \phi_k)\left(x + \frac{1}{2}L_i\right) \\ &= \phi_i * (\phi_j * \phi_k). \end{aligned} \tag{2.2}$$

Similarly to the field theory in noncommutative spacetime, the natural choice for Tr (trace) is the trace on the matrix values plus the integral over the spacetime. However, to satisfy the necessary cyclicity condition, the integrand is restricted to have a total zero dipole vector. More precisely, the integral serves as the proper Tr over the functions of zero dipole vector; i.e.

$$\int \phi_1 * \phi_2 * \cdots * \phi_n = \int \phi_n * \phi_1 * \cdots * \phi_{n-1} \tag{2.3}$$

with the condition that $\sum_{i=1}^{n} L_i = 0$, where L_i is the dipole vector for ϕ_i. As a result, together with the translational invariance, both the sum of the external momenta and the total dipole vector should vanish at each vertex.

We also define the complex conjugate of a field. Demanding $(\phi^\dagger * \phi)$ to be real, i.e.

$$\phi^\dagger * \phi = (\phi^\dagger * \phi)^\dagger , \tag{2.4}$$

fixes the dipole vector of $\phi^\dagger$ to be $-L$ when ϕ has dipole vector L. Therefore, the dipole vector of any real (hermitian) field (in particular, the gauge fields) is zero. As the dipole vector of $\phi^\dagger$ is fixed, we can show that

$$(\phi_i * \phi_j)^\dagger = \phi_j^\dagger * \phi_i^\dagger . \tag{2.5}$$

Furthermore, it can be shown that the usual derivative satisfies the Leibniz rule with respect to the star product, i.e.

$$\partial_\mu(\phi_1 * \phi_2) = (\partial_\mu \phi_1) * \phi_2 + \phi_1 * (\partial_\mu \phi_2) . \tag{2.6}$$

To formulate the field theory action, therefore, ∂_μ is used as the proper derivative to write down the kinetic terms of the DFTs.

3. Noncommutative dipole gauge theory with adjoint matter

Equipped with the above mathematics, we are ready to formulate DFTs. In general, the way to obtain the action of DFTs is to replace all the products in the actions of the ordinary commutative field theories with the dipole star products as in (2.1). Meanwhile, all the fields should be associated with the proper dipole vectors.

Here we restrict ourselves to the U(1) gauge theory with the scalar adjoint matter fields ϕ (with the dipole vector L). Other gauge groups can be studied in a similar way. The gauge fields (photons), being hermitian, should have a zero dipole vector. Thus, the pure gauge theory is exactly the same as the commutative theory and the propagator of the gauge field is unchanged. On the other hand, the gauge fields coupled to the matter fields reveal the dipole structure of the charged matter fields. The covariant derivative of the adjoint matter field is defined as:

$$\begin{aligned} D_\mu \phi &\equiv \partial_\mu \phi + ig(A_\mu * \phi - \phi * A_\mu) \\ &= \partial_\mu \phi + ig \left[A_\mu \left(x - \frac{L}{2} \right) \phi(x) - \phi(x) A_\mu \left(x + \frac{L}{2} \right) \right] , \end{aligned} \tag{3.1}$$

and the lagrangian is

$$\mathcal{L} = D_\mu \phi^\dagger * D^\mu \phi - m^2 \phi^\dagger * \phi - \frac{1}{4} F_{\mu\nu} * F^{\mu\nu} \tag{3.2a}$$

$$= D_\mu \phi^\dagger * D^\mu \phi - m^2 \phi^\dagger * \phi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}, \tag{3.2b}$$

which is invariant under the gauge transformation:

$$\phi \to U * \phi * U^{-1} \tag{3.3a}$$

$$A_\mu \to U * A_\mu * U^{-1} + \frac{i}{g} \partial_\mu U * U^{-1} = U A_\mu U^{-1} + \frac{i}{g} \partial_\mu U U^{-1}, \tag{3.3b}$$

where $U \in \mathrm{U}(1)$.

Now, we are ready to study the Feynman rules for the propagators and the vertices.

3.1 Propagators

In (3.2), the kinetic term of the gauge field is the same as the ordinary commutative theory, due to the fact that A_μ has zero dipole vector. Therefore, the propagator of A_μ is the same as the commutative QED. (See figure 1a.)

Now, we expand the term involving ϕ:

$$\begin{aligned} D_\mu \phi^\dagger * D^\mu \phi = &\left(\partial_\mu \phi^\dagger - ig\phi^\dagger * A_\mu + igA_\mu * \phi^\dagger\right) * (\partial^\mu \phi + igA^\mu * \phi - ig\phi * A^\mu) \\ =&\partial_\mu \phi^\dagger * \partial^\mu \phi + & (3.4a) \\ &+ ig\Big\{\partial_\mu \phi^\dagger * A^\mu * \phi - \partial_\mu \phi^\dagger * \phi * A^\mu - \\ &\quad - \phi^\dagger * A_\mu * \partial^\mu \phi + A_\mu * \phi^\dagger * \partial^\mu \phi\Big\} + & (3.4b) \\ &+ g^2\Big\{\phi^\dagger * A_\mu * A^\mu * \phi - \phi^\dagger * A_\mu * \phi * A^\mu - \\ &\quad - A_\mu * \phi^\dagger * A^\mu * \phi + A_\mu * \phi^\dagger * \phi * A^\mu\Big\}. & (3.4c) \end{aligned}$$

Consider

$$(\phi^\dagger * \phi)(x) = (\phi^\dagger \phi)\left(x - \frac{1}{2}L\right), \tag{3.5}$$

after integration which becomes

$$\int d^D x\, \partial_\mu \phi^\dagger * \partial^\mu \phi = \int d^D x\, \partial_\mu \phi^\dagger \partial^\mu \phi\,. \tag{3.6}$$

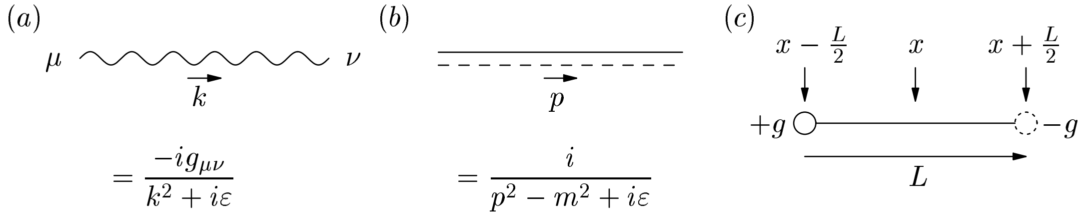


Figure 1: (a) Propagator of the photon field A. (b) Propagator of the adjoint matter field ϕ. (c) The dipole structure of ϕ with the positive- and negative-charged ends separated by L.

Hence, the dipole dependence in the kinetic term (3.4a) as well as that in the mass term $-m^2\phi^\dagger * \phi$ of (3.2) are removed. As a result, the propagator of ϕ is exactly the same as the commutative counterpart. (See figure 1*b*. The meaning of the doubled line will be explained soon.)

3.2 3-leg vertices

Next, we study the interactions of matter and gauge fields in (3.4b), which correspond to the 3-leg vertices. By the second line of (2.2) and a spacetime translation, the first term in (3.4b) becomes

$$\begin{aligned}(\partial_\mu\phi^\dagger * A^\mu * \phi)(x) &= \partial_\mu\phi^\dagger\left(x-\frac{L}{2}\right)A^\mu(x-L)\phi\left(x-\frac{L}{2}\right)\\ \text{under translation} &\to \partial_\mu\phi^\dagger(x)A^\mu\left(x-\frac{L}{2}\right)\phi(x),\end{aligned}\tag{3.7}$$

and the second term becomes

$$\begin{aligned}(-\partial_\mu\phi^\dagger * \phi * A^\mu)(x) &= -\partial_\mu\phi^\dagger\left(x-\frac{L}{2}\right)\phi\left(x-\frac{L}{2}\right)A^\mu(x)\\ \text{under translation} &\to -\partial_\mu\phi^\dagger(x)\phi(x)A^\mu\left(x+\frac{L}{2}\right).\end{aligned}\tag{3.8}$$

Similar results are obtained for $-\phi^\dagger * A_\mu * \partial^\mu\phi$ and $A_\mu * \phi^\dagger * \partial^\mu\phi$, and all the terms in (3.4b) are thus equivalent to

$$-igA^\mu\left(x-\frac{L}{2}\right)(\phi^\dagger\overleftrightarrow{\partial_\mu}\phi)(x)+igA^\mu\left(x+\frac{L}{2}\right)(\phi^\dagger\overleftrightarrow{\partial_\mu}\phi)(x)\,.\tag{3.9}$$

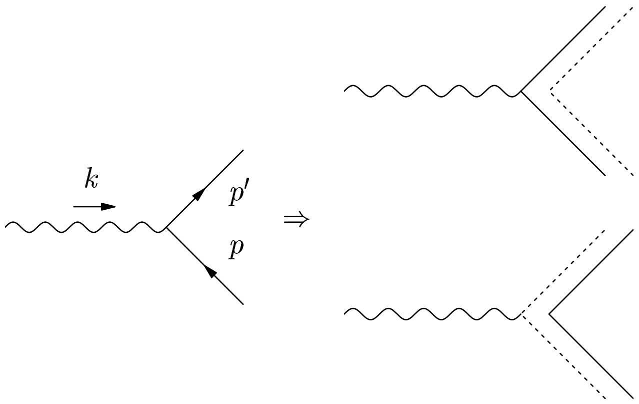
Each individual term in the above is identical to the 3-leg interaction in scalar electrodynamics except that the point where A^μ is coupled is shifted from x to $x \pm L/2$ with the charge $\mp g$ respectively. This has a natural interpretation in terms of the dipole structure: when $\phi(x)$ is centered at x, it has two ends with the opposite charges $\pm g$ at $x \mp L/2$. In the diagram, we use a doubled line to denote the propagator with the solid and dashed lines representing $+g$ and $-g$ endpoints. (See figure 1*c*.)

At the same time, A^μ can couple to ϕ at either the positive-charged or the negative-charged side. The shift of $\pm L/2$ will introduce an extra phase $e^{\pm ik\cdot L/2}$ in the momentum space, and the Feynman rules for the vertices are modified with this *dipole phase.* (See figure 2 for the Feynman rules.)

3.3 4-leg vertices

Similarly to the second line of (2.2), the star product of four fields behaves as

$$\begin{aligned}(\phi_i * \phi_j * \phi_k * \phi_l)(x) = \phi_i&\left(x-\frac{L_j+L_k+L_l}{2}\right)\phi_i\left(x+\frac{L_i-L_k-L_l}{2}\right)\times\\ &\times\phi_k\left(x+\frac{L_i+L_j-L_l}{2}\right)\phi_l\left(x+\frac{L_i+L_j+L_k}{2}\right).\end{aligned}\tag{3.10}$$



$$= -ig(p_\mu + p'_\mu)e^{-\frac{i}{2}k\cdot L}\times$$
$$\times(2\pi)^D\delta(p - p' + k)$$

$$= +ig(p_\mu + p'_\mu)e^{+\frac{i}{2}k\cdot L}\times$$
$$\times(2\pi)^D\delta(p - p' + k)$$

Figure 2: Feynman rules for 3-leg vertices.

Therefore, the four terms inside the curly bracket of (3.4c) are

$$\phi^\dagger\left(x-\frac{L}{2}\right)A_\mu(x-L)A^\mu(x-L)\phi\left(x-\frac{L}{2}\right)-\phi^\dagger\left(x-\frac{L}{2}\right)A_\mu(x-L)\phi\left(x-\frac{L}{2}\right)A^\mu(x)-$$
$$-A_\mu(x)\phi^\dagger\left(x-\frac{L}{2}\right)A^\mu(x-L)\phi\left(x-\frac{L}{2}\right)+A_\mu(x)\phi^\dagger\left(x-\frac{L}{2}\right)\phi\left(x-\frac{L}{2}\right)A^\mu(x) \tag{3.11}$$

and become

$$\phi^\dagger(x)A_\mu\left(x-\frac{L}{2}\right)A^\mu\left(x-\frac{L}{2}\right)\phi(x)-\phi^\dagger(x)A_\mu\left(x-\frac{L}{2}\right)\phi(x)A^\mu\left(x+\frac{L}{2}\right)-$$
$$-A_\mu\left(x+\frac{L}{2}\right)\phi^\dagger(x)A^\mu\left(x-\frac{L}{2}\right)\phi(x)+A_\mu\left(x+\frac{L}{2}\right)\phi^\dagger(x)\phi(x)A^\mu\left(x+\frac{L}{2}\right) \tag{3.12}$$

under the spacetime translation.

Again, the two legs of the gauge field can act on either end of the dipole field ϕ at $x \pm L/2$. The Feynman rules for the 4-leg vertices are almost the same as that in scalar electrodynamics, while the coupling g^2 is replaced by $(\pm g)(\pm g)$ and the dipole phases are also introduced. (See figure 3 for the Feynman rules.)

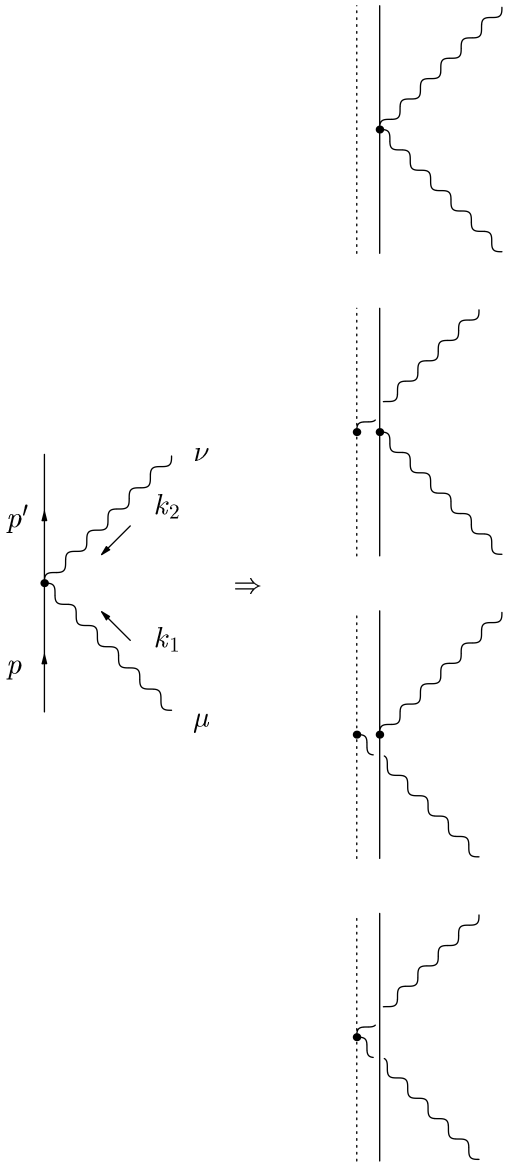
4. Unitarity of noncommutative dipole field theory

4.1 Optical theorem and unitarity

In this section, we examine one-loop diagrams of noncommutative dipole gauge theory to see if unitarity is satisfied.

One straightforward consequence of unitarity is the *optical theorem*, which states that the imaginary part of any scattering amplitude arises from a sum of contributions from all possible intermediate-state particles; i.e.

$$2\,\mathrm{Im}\,M(a\to b) = \sum_f M^*(b\to f)M(a\to f)\,, \tag{4.1}$$



$$= 2ig^2 g_{\mu\nu} e^{-\frac{i}{2}(k_1+k_2)\cdot L} \times$$
$$\times (2\pi)^D \delta(p - p' + k_1 + k_2)$$

$$= -2ig^2 g_{\mu\nu} e^{-\frac{i}{2}(k_1-k_2)\cdot L} \times$$
$$\times (2\pi)^D \delta(p - p' + k_1 + k_2)$$

$$= -2ig^2 g_{\mu\nu} e^{\frac{i}{2}(k_1-k_2)\cdot L} \times$$
$$\times (2\pi)^D \delta(p - p' + k_1 + k_2)$$

$$= 2ig^2 g_{\mu\nu} e^{\frac{i}{2}(k_1+k_2)\cdot L} \times$$
$$\times (2\pi)^D \delta(p - p' + k_1 + k_2)$$

Figure 3: Feynman rules for 4-leg vertices.

where the sum runs over all possible sets f of final-state particles and includes phase space integrations for each particle in f.

Quantum field theories actually satisfy more restrictive relation called the *cutting rule*, which is a generalization of the optical theorem to Feynman diagrams of all orders in perturbation theory. This states that the imaginary part of any Feynman diagram is given by the following algorithm:

1. Cut through the diagram into two separate pieces in all possible ways such that the cut propagators can simultaneously be put on shell.

2. For each cut, place the virtual particle on-shell by replacing the propagator with a

delta function:

$$\frac{1}{p^2 - m^2 + i\varepsilon} \to -2\pi i \delta(p^2 - m^2)\,. \tag{4.2}$$

3. Perform the integrals over p and sum over the contributions of all possible cuts.

The cutting rule is more general than the constraint of unitarity because it applies to the scattering amplitudes as well as the off-shell Green's functions. The detailed discussion can be found in many QFT textbooks [35].

As shown in [1], the spacetime noncommutative scalar field theory does not obey unitarity when there is a space-time noncommutativity ($\theta^{0i} \neq 0$), while in the case of spatial noncommutativity ($\theta^{0i} = 0$, $\theta^{ij} \neq 0$) the cutting rules are satisfied. Following [1], in our case of the DFT, we will first show that the two-point function of ϕ (self energy diagram) violates the usual cutting rules when the associated dipole vector L is *timelike*. On the other hand, when L is *spacelike*, the unitarity is satisfied. Later, we consider the $2\phi \to 2\phi$ scattering, and the same conclusion is made.

However, the diagrams with external photon legs signal no unitarity violation (at least at one-loop level) even for timelike L. This suggests that there might be some symmetry which keeps the unitarity in the limit of the string theory, but we will not explore this further.

The reason for the failure of unitarity is very similar to that of [1]. Since the Feynman rules for the vertices are manifestly real functions of momenta (typically of the form $\sim e^{ik\cdot L} + e^{-ik\cdot L}$), at first look, one would expect that the Feynman diagrams could develop a branch cut only when the internal lines go on-shell. This would imply that the imaginary parts of Feynman diagrams would be given by the same cutting rules as the ordinary commutative field theory and thus unitarity would be satisfied. However, a closer examination of the high energy behavior of the oscillatory factors that arise from the dipole phases shows that unitarity is broken. A Feynman integral can be defined via analytic continuation by Wick rotation, but the resulting amplitude will develop branch cuts when $L^2 > 0$ (timelike) back in Minkowski space. These additional branch cuts are responsible for the breakdown of the cutting rule.

For $L^2 = 0$ (lightlike), the amplitude is ill-defined because of the infrared divergence. A sensible theory should have infrared-safe observables and this may requires all order re-summation of infrared divergent terms in the perturbative series. We will not attempt to address this issue in this paper and focus on the amplitudes which do not suffer from infrared singularities. (See [1].)

4.2 Two-point function of ϕ

Now, we study the two-point functions of ϕ. The one-loop Feynman diagrams are listed in figure 4. Since ϕ has the dipole structure and A can act on either end of the dipole, the diagrams have planar and nonplanar parts. By the Feynman rules, the planar amplitude is:

$$iM_p = -2g^2 \int \frac{d^D k}{(2\pi)^D} \frac{g^{\mu\nu}}{k^2 + i\varepsilon} \frac{(2p_\mu + k_\mu)(2p_\nu + k_\nu)}{(p+k)^2 - m^2 + i\varepsilon}, \tag{4.3}$$

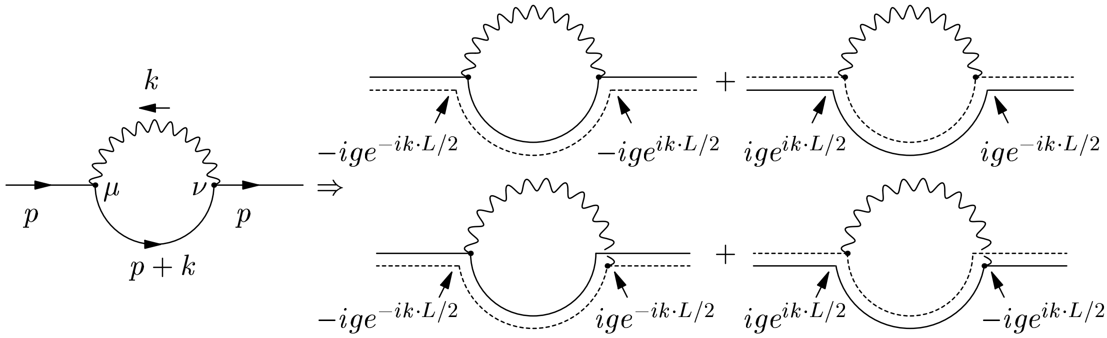


Figure 4: Feynman diagrams for the two-point function of ϕ. The two diagrams on the top are the planar part while those on the bottom are the nonplanar part. Each vertex contribution with the corresponding dipole phase is indicated.

and the nonplanar amplitude is:

$$iM_{np} = 2g^2 \int \frac{d^D k}{(2\pi)^D} \frac{g^{\mu\nu}}{k^2 + i\varepsilon} \frac{(2p_\mu + k_\mu)(2p_\nu + k_\nu)}{(p+k)^2 - m^2 + i\varepsilon} \cos k \cdot L \,, \tag{4.4}$$

where D is the spacetime dimension. We will focus on the nonplanar terms because it is obvious that the planar parts satisfy the unitarity just as the ordinary commutative field theory does.

First, we perform a Wick rotation to euclidean space by the usual analytic continuation $p^0 \to ip_E^2$, $p^2 \to -p_E^2$, $L = (L^0, \vec{L}) \to L_E = (-iL^0, \vec{L})$, $p \cdot L \to p_E \cdot L_E$:

$$M_{np} = -g^2 \int \frac{d^D k_E}{(2\pi)^D} \frac{-(k_E + 2p_E)^2}{k_E^2[(k_E + p_E)^2 + m^2]} (e^{ik_E \cdot L_E} + \text{c.c.}) \,, \tag{4.5}$$

and then combine the denominators by Schwinger parameters

$$\frac{1}{k_E^2[(k_E + p_E)^2 + m^2]} = \int_0^1 dx \int_0^\infty d\alpha \, \alpha \, e^{-\alpha x k_E^2} e^{-\alpha(1-x)[(k_E + p_E)^2 + m^2]} \tag{4.6}$$

to obtain

$$M_{np} = -g^2 \int \frac{d^D k_E}{(2\pi)^D} \int_0^1 dx \int_0^\infty d\alpha \, \alpha (k_E + 2p_E)^2 e^{-\alpha x k_E^2 - \alpha(1-x)[(k_E+p_E)^2 + m^2] + ik_E \cdot L_E} + \text{c.c.} \tag{4.7}$$

Consider the exponent in (4.7):

$$\begin{aligned} &\alpha x k_E^2 + \alpha(1-x)[(k_E + p_E)^2 + m^2] - ik_E \cdot L_E = \\ &\quad = \left\{ \alpha \left[k_E + (1-x)p_E - \frac{i}{2\alpha} L_E \right]^2 + x(1-x)p_E^2 + \right. \\ &\qquad \left. + (1-x)m^2 + \frac{L_E^2}{4\alpha^2} + \frac{i(1-x)}{\alpha} p_E \cdot L_E \right\} \qquad (4.8) \\ &\quad \equiv \alpha \left\{ k_E'^2 + x(1-x)p_E^2 + (1-x)m^2 + \frac{L_E^2}{4\alpha^2} + \frac{i(1-x)}{\alpha} p_E \cdot L_E \right\} . \qquad (4.9) \end{aligned}$$

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By a change of variable from k_E to k'_E, (4.7) becomes

$$M_{np} = -\, g^2 \int \frac{d^D k'_E}{(2\pi)^D} \int_0^1 dx \int_0^\infty d\alpha\, \alpha \left(k'_E + (x+1)p_E + \frac{i}{2\alpha} L_E\right)^2 \times \\ \times e^{-\alpha\left[k'^2_E + x(1-x)p_E^2 + (1-x)m^2 + \frac{L_E^2}{4\alpha^2} + \frac{i(1-x)}{\alpha} p_E \cdot L_E\right]} + \text{c.c.}\,, \tag{4.10}$$

where only k'^2_E and $[(x+1)p_E + \frac{i}{2\alpha} L_E]^2$ in the first line contribute due to the spherical symmetry in k'_E space. After integrating over dk'_E, (4.10) becomes

$$M_{np} = -\, \frac{g^2}{2^D \pi^{D/2}} \int_0^1 dx \int_0^\infty d\alpha\, \alpha \left\{ \frac{D/2}{\alpha^{(D+2)/2}} + \frac{\left[(x+1)p_E + \frac{i}{2\alpha} L_E\right]^2}{\alpha^{D/2}} \right\} \times \\ \times e^{-\alpha\left[x(1-x)p_E^2 + (1-x)m^2 + \frac{L_E^2}{4\alpha^2} + \frac{i(1-x)}{\alpha} p_E \cdot L_E\right]} + \text{c.c.} \tag{4.11}$$

In particular, we can work out the integration over $d\alpha$ for $D = 3$:

$$M_{np,D=3} = \frac{g^2}{4\pi} \int_0^1 dx \frac{e^{-\sqrt{(1-x)(m^2+p_E^2 x)L_E^2}}}{\sqrt{L_E^2}} \times \\ \times \Bigg\{ -\cos[(x-1)p_E \cdot L_E] \times \\ \times \left(2 + \frac{\sqrt{L_E^2}}{\sqrt{(1-x)(m^2+p_E^2 x)}} [(x-1)m^2 + p_E^2(2x^2+x+1)] \right) + \\ + \sin[(x-1)p_E \cdot L_E](2(1+x)p_E \cdot L_E) \Bigg\}. \tag{4.12}$$

This is very complicated, but it is greatly simplified if we consider the on-shell condition ($p^2 = m^2$):

$$M_{np,D=3} = \frac{g^2}{4\pi} \int_0^1 dx \frac{e^{-(1-x)m\sqrt{-L^2}}}{\sqrt{-L^2}} \left\{ \frac{2}{1-x} [m(x^2+1)\sqrt{-L^2} + x - 1] \cos((x-1)p \cdot L) + \\ + 2p \cdot L(1+x) \sin((x-1)p \cdot L) \right\}, \tag{4.13}$$

where we have analytically continued back to the Minkowski space.

For $L^2 < 0$ (spacelike), obviously (4.13) gives $\operatorname{Im} M_{np} = 0$. Meanwhile, the right hand side of (4.1) is zero because the process that an on-shell massive particle decays into an on-shell massive particle plus a real photon is kinematically forbidden. Therefore, unitarity is satisfied.

On the other hand, for $L^2 > 0$ (timelike), $\sqrt{-L^2} = i\,|L|$ and (4.13) gives $\operatorname{Im} M \neq 0$, while the right hand side of (4.1) is still zero. Therefore, we find that unitarity is violated when the dipole vector L is timelike (at least for the on-shell condition).

4.3 $2\phi \to 2\phi$ scattering amplitude

Next, we consider the $2\phi \to 2\phi$ scattering amplitude. The one-loop Feynman diagrams are shown in figure 5. For each 4-leg vertex, we have an extra dipole phase depending on which ends the two virtual photons act on. Consequently, the total contribution due to the vertices with the dipole phases is

$$
\begin{aligned}
&2i\left(ge^{\frac{i}{2}k\cdot L} - ge^{-\frac{i}{2}k\cdot L}\right)\left(ge^{\frac{i}{2}(s-k)\cdot L} - ge^{-\frac{i}{2}(s-k)\cdot L}\right)\times\\
&\times 2i\left(ge^{-\frac{i}{2}k\cdot L} - ge^{\frac{i}{2}k\cdot L}\right)\left(ge^{-\frac{i}{2}(s-k)\cdot L} - ge^{+\frac{i}{2}(s-k)\cdot L}\right) =\\
&\quad = -16g^4\left(1-\cos k\cdot L - \cos(s-k)\cdot L + \cos(k\cdot L)\cos(s-k)\cdot L\right), \qquad (4.14)
\end{aligned}
$$

where $s = p_1 + p_2$.

As shown in figure 5, the intermediate loop is made of photon propagators, which have no dipole structure, so the "non-planarity" is no longer a proper way to describe the non-triviality. Nevertheless, we can classify the diagrams according to the way in which the dipole phase is coupled to the internal momentum k of the loop. In figure 5a, the dipole phases are totally decoupled from the internal momentum and therefore unitarity of those diagrams is guaranteed as the ordinary commutative theory is unitary. The dipole phases from the diagrams in figure 5a are the "trivial" part of (4.14). On the other hand, the diagrams in figure 5b 5c altogether give the nontrivial amplitude[1]

$$
\begin{aligned}
iM_{nt} = \frac{1}{2}g^4 g_{\mu\nu}g_{\rho\sigma}\int \frac{d^Dk}{(2\pi)^D}\frac{-ig^{\mu\rho}}{k^2+i\varepsilon}\frac{-ig^{\nu\sigma}}{(s-k)^2+i\varepsilon}\times\\
\times\left\{16\cos k\cdot L + 16\cos(s-k)\cdot L - 8\cos(s-2k)\cdot L\right\}, \qquad (4.15)
\end{aligned}
$$

where the factor $1/2$ is due to the internal bosonic loop.

Following the same procedure in the previous subsection, in euclidean space, we have

$$
\begin{aligned}
M_{nt} = &\frac{-4D[1+(-1)^D]g^4}{2^{D-1}\pi^{D/2}}\int_0^1 dx\int_0^\infty d\alpha\,\alpha^{1-D/2}e^{-\alpha\left[(x(1-x)s_E^2+\frac{L_E^2}{4\alpha^2}\right]}\cos((1-x)s_E\cdot L_E)+\\
&+\frac{2Dg^4}{2^{D-1}\pi^{D/2}}\int_0^1 dx\int_0^\infty d\alpha\,\alpha^{1-D/2}e^{-\alpha\left[(x(1-x)s_E^2+\frac{L_E^2}{\alpha^2}\right]}\cos((1-2x)s_E\cdot L_E). \qquad (4.16)
\end{aligned}
$$

We evaluate this integral for $D = 3$ and 4. Back to Minkowski space, (4.16) gives

$$
M_{nt,D=3} = \frac{3g^4}{2\pi}\int_0^1 dx\,\frac{e^{-2\sqrt{-L^2}\sqrt{-s^2}\sqrt{(1-x)x}}}{\sqrt{-s^2}\sqrt{(1-x)x}}\cos((1-2x)s\cdot L), \qquad (4.17a)
$$

$$
\begin{aligned}
M_{nt,D=4} = &-\frac{8g^4}{\pi^2}\int_0^1 dx\,K_0\left(\sqrt{-L^2}\sqrt{-s^2(1-x)x}\right)\cos((1-x)s\cdot L)+\\
&+\frac{2g^4}{\pi^2}\int_0^1 dx\,K_0\left(2\sqrt{-L^2}\sqrt{-s^2(1-x)x}\right)\cos((1-2x)s\cdot L), \qquad (4.17b)
\end{aligned}
$$

[1] In fact, the diagrams in figure 5a can all be drawn on the surface of a sphere although some of them are non-planar, while those in figure 5b 5c can be drawn on the surface of a torus. Therefore, instead of non-planarity, we can use *genus* to classify the non-triviality in all cases.

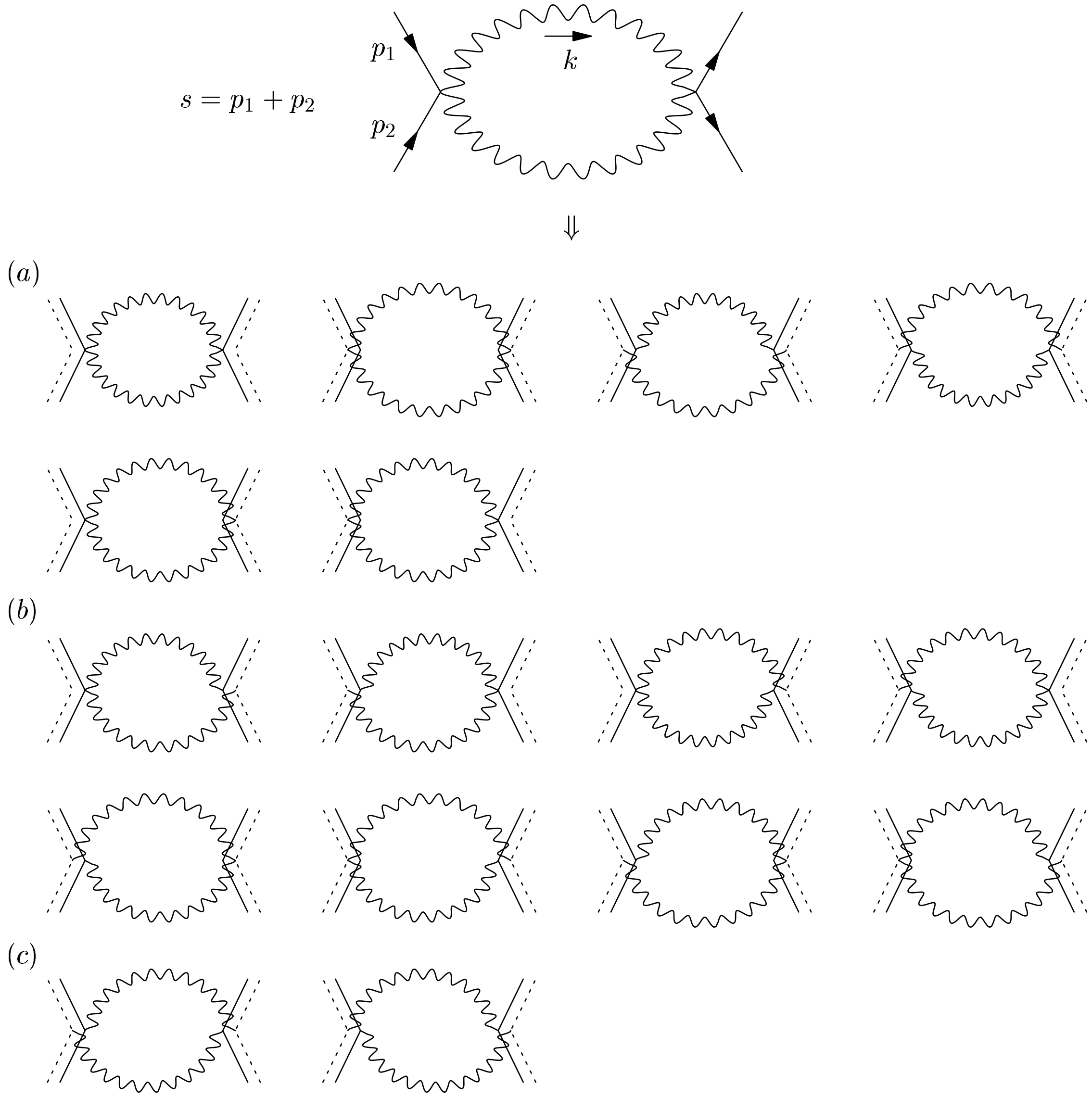

Figure 5: Feynman diagrams for $2\phi \to 2\phi$ process: The diagrams of (a) give the vertex contribution $-4g^4(1{+}1{+}1{+}1{+}e^{is\cdot L}{+}e^{-is\cdot L}) = -8g^4(2{+}\cos s{\cdot}L)$. Those of (b) give $16g^4(\cos k{\cdot}L{+}\cos(s{-}k){\cdot}L)$, and those of (c) give $-4g^4(e^{-i(s-2k)\cdot L} + e^{i(s-2k)\cdot L}) = -8g^4\cos(s-2k)\cdot L$. All together we get the result in (4.14).

where K_0 is the modified Bessel function of the second kind. (4.17) is complicated and hard to be compared with the right hand side of (4.1). However, if we focus on the situation for $s^2 < 0$ (spacelike), the right hand side of (4.1) vanishes because energy-momentum conservation forbids two particles (whether on-shell or off-shell) with spacelike s to scatter into two real photons.

Assume $s^2 < 0$. For $L^2 < 0$ (spacelike), obviously (4.17) gives $\operatorname{Im} M_{np} = 0$ and so unitarity is satisfied. For $L^2 > 0$ (timelike), however, (4.17) leads to $\operatorname{Im} M_{np} \neq 0$ and therefore unitarity is violated.

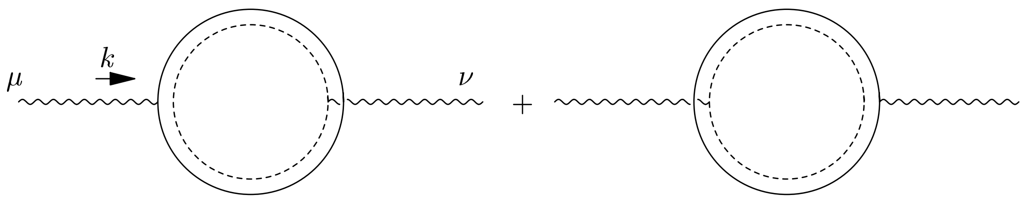

Figure 6: Nonplanar one-loop Feynman diagrams for vacuum polarization.

4.4 Scattering with external photon legs

Finally, we study the unitarity for the diagrams with external photon legs. Consider the vacuum polarization diagram in figure 6. The amplitude for the nonplanar part is given by

$$iM_{np} = -2g^2 \int \frac{d^D l}{(2\pi)^D} \frac{\epsilon_\mu \epsilon^*_\nu (k_\mu + 2l_\mu)(k_\nu + 2l_\nu)}{(l^2 - m^2 + i\varepsilon)((l+k)^2 - m^2 + i\varepsilon)} \cos k \cdot L\,. \tag{4.18}$$

The dipole phase $\cos k \cdot L$ in (4.18) is real and completely decoupled from the internal momentum l. Therefore, the resulting amplitude will not give a new branch cut and the imaginary part of the amplitude will be given by the same cutting rule as the ordinary commutative field theory. Thus, unitarity is always satisfied up to one-loop level no matter L is spacelike, timelike, or even lightlike.

At one-loop level, the same conclusion can be made for $2\gamma \to 2\gamma$ processes as well. However, the unitarity might still be violated at the two-loop or higher-loop level since the dipole phases will be coupled with the internal momenta at the higher-loop level. The fact that the unitarity is satisfied at one-loop level whenever the external legs of the diagram are all photons suggests that some symmetry in the theory saves the unitarity and should be understood in the limit of the string theory realization. We defer this issue to our further study.

5. A nonlocal quantum mechanics

Violation of unitarity also implies that some energy levels get an imaginary part. This is easiest to analyze for 0+1D systems. In noncommutative geometry, spacetime must be at least two dimensional, but the nonlocal dipole theories can be defined with no space and only time. We will now explore this manifestation of nonunitarity.

We will discuss two theories. The first is a harmonic oscillator with the nonlocal action

$$S = \frac{1}{2} \int \left\{ \dot{x}(t)^2 + x(t-T)x(t+T) \right\} dt\,, \qquad x : (-\infty, \infty) \mapsto \mathbb{R} \tag{5.1}$$

and the second is a pair of local harmonic oscillators deformed by a nonlocal interaction term:

$$S = \frac{1}{2} \int \left\{ |\dot{z}(t)|^2 + |z(t)|^2 + \lambda |z(t)|^4 - \lambda |z(t+T)|^2 |z(t-T)|^2 \right\} dt\,, \qquad z : (-\infty, \infty) \mapsto \mathbb{C}. \tag{5.2}$$

Here t is real and z is complex and we set $\hbar = 1$, $m = 1$ and $\omega = 1$. The second action is the action of a dipole theory with a complex field z of dipole length $2T$ and an interaction term proportional to $(z * z^\dagger - z^\dagger * z)^2$.

How will we find the energy levels? Because of the time nonlocality, a hamiltonian formulation is rather cumbersome (see [28] for details). Instead, we will adopt the following approach. We compactify time on a circle of radius R (in euclidean time) and expand the partition function as

$$Z(R) = \sum_n C_n e^{-2\pi R E_n} . \tag{5.3}$$

Because of nonunitarity, there is no guarantee that $C_n = 1$, as we will see. The energy levels can be read off as the poles of the Fourier transform of $Z'(R)/Z(R)$ as shown below.

5.1 Nonlocal harmonic oscillator

In the compactified euclidean time, the action (5.1) can be written as

$$S = \pi R \sum_{n=-\infty}^{\infty} |q_n|^2 \left\{ \left(\frac{n}{R}\right)^2 + e^{\frac{-2inT}{R}} \right\} \tag{5.4}$$

in terms of the Fourier transformed modes of $x(t)$: $q_n = q^*_{-n} = \frac{1}{2\pi R}\int_0^{2\pi R} x(t) e^{-int/R} dt$. The partition function is then obtained by path integral over the momentum space:

$$Z(R) \propto \int \prod_{n=-\infty}^{\infty} dq_n \, e^{-2\pi R S} \propto \int \prod_{n=1}^{\infty} dq_n dq_n^* \, e^{-\pi R \sum_{n=1}^{\infty} |q_n|^2 \left\{ 2\left(\frac{n}{R}\right)^2 + e^{\frac{-2inT}{R}} + e^{\frac{2inT}{R}} \right\}} . \tag{5.5}$$

By $\int_{-\infty}^{\infty} dz dz^* e^{-\alpha |z|^2} = \pi/\alpha$, the result reads as

$$Z(R) = \frac{1}{2\pi R \prod_{n=1}^{\infty} \left(1 + \frac{R^2}{n^2} \cos\frac{2nT}{R}\right)} , \tag{5.6}$$

where the overall constant factor is fixed such that when $T = 0$, $Z(R) = 1/(e^{\pi R} - e^{-\pi R}) = \sum_{n=0}^{\infty} e^{-2\pi R(n+1/2)}$ as it should be.

If $Z(R)$ has an expansion of the form (5.3), then

$$\log Z(R) = \log C_0 - 2\pi E_0 R + \frac{C_1}{C_0} e^{-2\pi(E_1 - E_0)R} + \frac{C_2}{C_0} e^{-2\pi(E_2 - E_0)R} + \cdots , \tag{5.7}$$

where $(\cdots)$ are higher exponentials which are products of the $e^{-2\pi(E_n - E_0)R}$ terms. The derivative of $\log Z(R)$ gives

$$\frac{Z'(R)}{Z(R)} = 2\pi \left\{ -E_0 - \sum_{n=1}^{\infty} \frac{C_n}{C_0} (E_n - E_0) e^{-2\pi(E_n - E_0)R} + \cdots \right\} . \tag{5.8}$$

The Fourier transform of the equation above is

$$\hat{Z}(\xi) \equiv \int_0^{\infty} e^{-2\pi i \xi R} \frac{Z'(R)}{Z(R)} dR = \frac{iE_0}{\xi} + \sum_{n=1}^{\infty} \frac{C_n}{C_0} \frac{i(E_n - E_0)}{\xi - i(E_n - E_0)} + \cdots , \qquad \text{with } \operatorname{Im}\xi < 0 , \tag{5.9}$$

from which the energy spectrum can be read off as the poles of $\hat{Z}(\xi)$ for $\xi \in \mathbb{C}$.

On the other hand, directly from (5.6), we get

$$\frac{Z'(R)}{Z(R)} = -\frac{1}{R} - \sum_{n=1}^{\infty} \frac{2R\cos\frac{2nT}{R} + 2nT\sin\frac{2nT}{R}}{n^2 + R^2\cos\frac{2nT}{R}}\,. \tag{5.10}$$

We can use Poisson summation formula to write

$$\begin{aligned}\frac{Z'(R)}{Z(R)} &= -\int_0^\infty \frac{2R\cos\frac{2xT}{R} + 2xT\sin\frac{2xT}{R}}{x^2 + R^2\cos\frac{2xT}{R}}\,dx - \\ &\quad -2\sum_{n=1}^{\infty}\int_0^\infty \frac{2R\cos\frac{2xT}{R} + 2xT\sin\frac{2xT}{R}}{x^2 + R^2\cos\frac{2xT}{R}}\cos(2\pi n x)\,dx \\ &= -\int_0^\infty \frac{2\cos 2xT + 2xT\sin 2xT}{x^2 + \cos 2xT}\,dx - \\ &\quad -2\sum_{n=1}^{\infty}\int_0^\infty \frac{2\cos 2xT + 2xT\sin 2xT}{x^2 + \cos 2xT}\cos(2\pi n R x)\,dx\,. \end{aligned} \tag{5.11}$$

With $\int_0^\infty \cos(2\pi n R x)e^{-2\pi i\xi R}dR = i\xi/2\pi(n^2x^2 - \xi^2)$, we have the Fourier transform of (5.11):

$$\begin{aligned}\hat{Z}(\xi) &= -\frac{1}{2\pi i\xi}\int_0^\infty \frac{2\cos 2xT + 2xT\sin 2xT}{x^2 + \cos 2xT}\,dx - \\ &\quad -\sum_{n=1}^{\infty}\int_{-\infty}^{\infty} \frac{2\cos 2zT + 2zT\sin 2zT}{z^2 + \cos 2zT}\,\frac{i\xi}{2\pi(nz - \xi)(nz + \xi)}\,dz\,. \end{aligned} \tag{5.12}$$

To evaluate the second term of (5.12), consider the contour enclosing the upper half complex plane. The poles $-\xi/n$ are inside the contour while the poles ξ/n are outside (remember $\operatorname{Im}\xi < 0$). By residue theorem, the residues at $-\xi/n$ give

$$\hat{Z}(\xi) = \frac{i}{\xi}E_0(T) - \sum_{n=1}^{\infty}\frac{n\cos\frac{2\xi T}{n} + \xi T\sin\frac{2\xi T}{n}}{\xi^2 + n^2\cos\frac{2\xi T}{n}} + \cdots\,, \tag{5.13}$$

where

$$E_0(T) \equiv \frac{1}{2\pi}\int_0^\infty \frac{2\cos 2xT + 2xT\sin 2xT}{x^2 + \cos 2xT}\,dx, \tag{5.14}$$

and $(\cdots)$ are the residues contributed from the singularities which satisfy $z^2 + \cos 2zT = 0$.

Comparing (5.13) with (5.9), we find the ground state energy is $E_0 = E_0(T)$, which reduces to $1/2$, the ground state energy of local harmonic oscillators, as $T = 0$. Furthermore, when $T = 0$, the pole at $\xi = in$ of (5.13) should match the pole $\xi = i(E_n - E_0)$ of (5.9). This gives $E_n = E_0 + n$ as the spectrum of local harmonic oscillators should be.

When $T \neq 0$, $i(E_n - E_0)$ will be the solutions to

$$\xi^2 + n^2\cos\frac{2\xi T}{n}\,, \qquad \operatorname{Im}\xi \geq 0\,. \tag{5.15}$$

If we interpret $f = (E_n - E_0)/n = -i\xi/n$ as the frequency of the classical solution, (5.15) is exactly the same as eq. (56) in [28], which is derived from the classical equation of motion of a nonlocal harmonic oscillator. Following [28], the graphical analysis shows f has complex solutions to (5.15).[2] Therefore, some excited energy levels are complex-valued when $T \neq 0$, which implies the violation of unitarity.

5.2 Local harmonic oscillator with a nonlocal perturbation

Now we work with the action (5.2). We treat the action as a 0+1D field theory and calculate the self-energy correction to the propagator of z. As shown in figure 7, the one-loop diagram is given by

$$iM(E) = \frac{\lambda}{2} \int_0^\infty \frac{dE'}{E'^2 - 1 + i\varepsilon} \left(2 - e^{i(E-E')T} - e^{-i(E-E')T}\right) \tag{5.16}$$

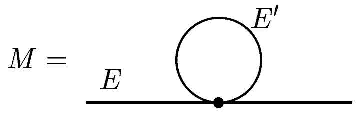

Figure 7: Self-energy correction.

with the dipole phases. Under Wick rotation, we get

$$\begin{aligned} M(E_E) &= -\lambda \int_0^\infty \frac{dE'_E}{E'^2_E + 1} \left[\left(1 - \cos((E_E - E'_E)\, T_E)\right]\right. \\ &= -\frac{\lambda\pi}{2} + \frac{\lambda}{2} \sin(E_E T_E) \left\{-2\,\mathrm{Ci}(iT_E) \sinh T_E + [2\,\mathrm{Si}(iT_E) + i\pi] \cosh T_E\right\}, \end{aligned} \tag{5.17}$$

where $\mathrm{Ci}(z)$ and $\mathrm{Si}(z)$ are cosine and sine integrals. Back to the Minkowski space, we have

$$M(E) = -\frac{\lambda\pi}{2} + i\frac{\lambda}{2} \sin(ET) \left\{2\,\mathrm{Ci}(T) \sin T + [\pi - 2\,\mathrm{Si}(T)] \cos T\right\} \tag{5.18}$$

and $\mathrm{Im}\, M \neq 0$.

However, the diagram in figure 7 cannot be cut into two separate pieces as described in subsection 4.1. This means the right hand side of (4.1) is zero. The cutting rule is thus violated with $\mathrm{Im}\, M \neq 0$ on the left hand side of (4.1), and therefore the theory is nonunitary.

Another way to understand the nonunitarity is to consider the propagator. The propagator with one-loop correction is given by:

$$G(E) = \frac{1}{E^2 - 1 - M(E) + i\varepsilon}, \tag{5.19}$$

which has a pole with an imaginary part. In field theory, the imaginary part of the pole indicates that the particle is unstable and can decay into lighter particles (see [35] again). However, $z(t)$ is the only field we have, so the particle simply decays into nothingness, which implies nonunitarity.

6. Discussion

In this paper, we have found that both $\phi \to \phi$ and $2\phi \to 2\phi$ processes are nonunitary at one-loop level when the dipole vector L is timelike. On the other hand, the processes in which the external legs are all photons are unitary at one-loop level regardless of the signature of L.

[2]The complex solutions to the frequency lead to the instability for classical systems with nonlocality of finite extent. This is discussed in [28].

DFTs can be constructed as an appropriate effective description of a low energy limit of string theory. In this limit, all the massive open string states are decoupled from the closed strings and the relevant degrees of the freedom are the massless open strings. String theory in this limit can be appropriately described in terms of DFT and thus the field theory should be unitary.

However, when the dipole vector is timelike, there is no regime in which DFT is an appropriate description of string theory and massive open string states cannot be neglected. This suggests the violation of unitarity in the field theory when L is timelike as we have found.

As unitarity of NCSYM can be restored [8, 9, 10], by adding an infinite tower of massive fields, we should have the NCOS completion for DFT to a unitary theory [34].

We also found that some energy levels or the poles of the propagator of 0+1D quantum mechanical systems with nonlocal interaction with finite extent in time have an imaginary part and therefore unitarity is also violated.

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